647510 **BSc6MatC603x** **Seat No :________**

B.Sc. Semester - 6 (*CBCS*) Examination
March/April-2023 (NEW COURSE)
Optimization and Numerical Analysis-2 (Core (New))

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q.1(A) Answer the following question. (Any Two) **(10)**

1. Define: Basic feasible solution. Explain the steps of two phase method.
2. Solve the following L.P.P. by simplex method.

$$\text{Max. } Z = 11x_1 + 9x_2$$
$$\text{Subject to} \quad 3x_1 + 2x_2 \le 8$$
$$2x_1 + 3x_2 \le 7$$
$$\text{and} \quad x_1, x_2 \ge 0.$$

3. Solve the following L.P.P. by Big-M Method.

$$\text{Min. } Z = 2x_1 + x_2$$
$$\text{Subject to} \quad 3x_1 + x_2 = 3$$
$$4x_1 + 3x_2 \ge 6$$
$$x_1 + 2x_2 \le 4$$
$$\text{and} \quad x_1, x_2 \ge 0.$$

Q.1(B) Answer the following questions. (Any One) **(04)**

1. Define: Convex and concave function. Explain canonical form of an L.P.P.
2. Solve the following L.P.P. by graphical method.

$$\text{Max. } Z = 20x + 10y$$
$$\text{Subject to} \quad x + 2y \le 40,$$
$$3x + y \ge 30$$
$$4x + 3y \ge 60$$
$$\text{and} \quad x, y \ge 0.$$

Q.2(A) Answer the following question. (Any Two) **(10)**

1. Explain Vogel's approximation method (V.A.M.) to find initial solution of a transportation problem.
2. Find optimal basic feasible solution by MODI method.

	P	**Q**	**R**	**S**	**Supply**
A	8	9	3	18	18
B	9	11	5	10	20
C	3	8	7	9	18
Demand	15	16	12	13	

3. Explain rules for constructing the dual from primal. Obtain the dual of the following L.P.P.

$$\text{Max. } Z_x = 2x_1 + 3x_2 + x_3$$
$$\text{Subject to} \quad 4x_1 + 3x_2 + x_3 = 6$$
$$x_1 + 2x_2 + 5x_3 = 4$$
$$\text{and} \quad x_1, x_2, x_3 \ge 0$$

Q.2(B) Answer the following questions. (Any One) **(04)**

1. Solve the following assignment problem.

	MEN				
		A	**B**	**C**	**D**
JOB	**1**	12	30	21	15
	2	18	33	9	31
	3	44	25	24	21
	4	23	30	28	24

2. Find the initial solution of the following transportation problem by lowest cost method.

	D_1	D_2	D_3	D_4	**Supply**
S_1	1	2	1	4	30
S_2	3	3	2	1	50
S_3	4	2	5	9	20
Demand	20	40	30	10	

Q.3(A) Answer the following question. (Any Two) **(10)**

1. State Gauss forward interpolation formula and hence derive Laplace Everett's interpolation formula.
2. Using Sterling's formula, find y_{45}, given that

x	10	20	30	40	50	60	70
y	600	512	439	346	243	133	32

3. Following data gives the percentage of criminals for different age groups:

Age (less than x)	25	30	40	50
% of criminals	52	67.3	84.1	94.4

Use Lagrange's formula to find the percentage of criminals under the age of 35.

Q.3(B) Answer the following questions. (Any One) **(04)**

1. If $f(x) = x^{-1}$, show that $f(x_0, x_1, \dots, x_n) = \frac{(-1)^n}{x_0\, x_1 \dots\, x_n}$.
2. Prove that the nth divided differences of a polynomial of nth degree are constants.

Q.4(A) Answer the following question. (Any Two) **(10)**

1. Derive formula of derivatives i.e. $\frac{dy}{dx}, \frac{d^2y}{dx^2}$ by using Gregory-Newton's forward interpolation formula.
2. The following table gives the results of an observation. θ is the observed temperature in degree centigrade of used cooling water, t is the time in minutes from the beginning of observation.

t	1	3	5	7	9	11
θ	85.3	74.5	67.0	60.5	54.3	50.1

Find approximately, the rate of cooling when t =10 using Newton's backward difference formula.

3. Derive Simpson's $\frac{3}{8}$ rule to evaluate $\int_a^b f(x)dx$.

Q.4(B) Answer the following questions. (Any One) (04)

1. Obtain the value of $f'(90)$ from the following data by using Sterling's formula correct up to 4 decimal places.

x	60	75	90	105	120
f(x)	28.2	38.2	43.2	40.9	37.7

2. By taking 10 partitions, evaluate: $\int_0^{10} \frac{dx}{1+x^2}$ to 6 decimal places by using

   (i) Trapezoidal rule and **(ii)** Simpson's $\frac{1}{3}$ rule.

Q.5(A) Answer the following question. (Any Two) (10)

1. Derive the formula to solve the differential equation $\frac{dy}{dx} = f(x,y),\ y(x_0) = y_0$ by Picard's method and specify its termination criteria.
2. Solve the differential equation $\frac{dy}{dx} = 1 + xy^2,\quad y(0) = 1,$ $y(0.1) = 1.105,\ y(0.2) = 1.223,\quad y(0.3) = 1.355$ by Milne's method to obtain $y(0.4)$.
3. Find the value of y at x=0.2 by Taylor's method, correct up to 3 decimal places, given that $y' = 2y + 3e^x,\ y(0) = 1$. Take $h = 0.1$.

Q.5(B) Answer the following questions. (Any One) (04)

1. Find the value of y at $x = 1$ correct up to 4 decimal places by Euler's method by taking step length of 0.2, given that $\frac{dy}{dx} = 2x + y,\ y(0) = 1.$
2. Solve the differential equation $\frac{dy}{dx} = \frac{1}{x+y}$, $y(0) = 1,$ at $x = 0.5$ by Runge-Kutta's method of 3rd order. Take $h = 0.25$.

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